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# De Moivre's Theorem

If $z_1 = x_1 + iy_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

and $z_2 = x_2 + iy_2$

$$= r_2(\cos\theta_2 + i\sin\theta_2)$$

we can show that

$$z_1 z_2 = r_1 r_2 \{\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\} \quad \text{--- (1)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)$$

A generalization of (1) leads to

$$z_1 z_2 \text{ --- } z_n = r_1 r_2 r_3 \text{ --- } r_n \left(\cos(\theta_1+\theta_2+ \text{ --- } +\theta_n) + i\sin(\theta_1+\theta_2+ \text{ --- } +\theta_n)\right)$$

and if $z_1 = z_2 = \text{ --- } = z_n = z$

this becomes

$$z^n = (r(\cos\theta + i\sin\theta))^n$$

$$= r^n(\cos n\theta + i\sin n\theta)$$

which is often called De Moivre's theorem.

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## Roots of C.N

A number $\omega$ is called an $n^{th}$ root of a complex number $z$ if $\omega^n = z$ and we write $\omega = z^{1/n}$. From De Moivre's theorem, we can ~~show~~ show that if $n$ is a positive integer

$$z^{1/n} = \{r(\cos\theta + i\sin\theta)\}^{1/n}$$

$$= r^{1/n}\left\{\cos\left(\frac{\theta+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right\}$$

from which it follows that there are $n$ different values for $z^{1/n}$ i.e. $n$ different $n^{th}$ root of $z$ provided $z \neq 0$

## Euler's formula

By assuming that the infinite series expansion $e^x = 1 + x + x^2/2! + x^3/3! + \ldots$ of elementary calculus holds when $x = i\theta$ we can arrive at the result

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$$e^{i\theta} = \cos\theta + i\sin\theta.$$

which is called Euler's formula.
It is more convenient, however

$$e^{z} = e^{x+iy} = e^{x} \cdot e^{iy} = e^{x}(\cos y + i\sin y)$$

In the special case where $y = 0$ this reduces to $e^{x}$

$$\left(e^{i\theta}\right)^{n} = e^{in\theta}.$$ using De Moivre's theorem.